Test questions: **Curve Sketching**

There are two sections to this test, with and without graphing calculator. BOTH sections should be given.

No Calculator Section

1. Determine the interval(s) on which the function $f(x) = x + 2\sin(x), 0 \le x \le 2\pi$ is increasing and on which it is decreasing. Show the work that leads to your answer. [5 points]

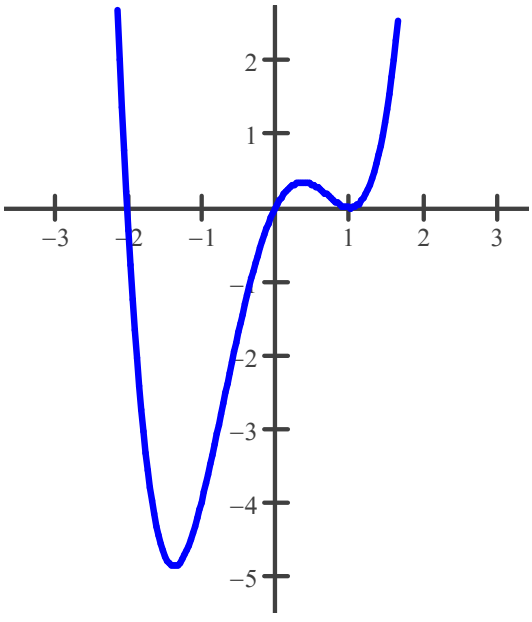
2. Determine the interval on which the function $g(x) = x^4 - 8x^3 - 30x^2 + 12$ is concave downwards. Show the work that leads to your answer. [5 points]

3. Write the equation of the line tangent to $h(x) = \cos(x) - x$ at the point $(0,1)$. Near the point of tangency does the tangent line lie above or below the graph? Justify your answer. [5 points]

4. Let f be the function whose derivative is given by $f'(x) = \frac{8}{x} - x^2$. On which of the following intervals is the function increasing? [3 points]

   a. $(-\infty, 0)$ only b. $(-\infty, -2)$ c. $(0, 2)$ only d. $(2, \infty)$ e. $(0, \infty)$

5. The second derivative of the function g is given by $g''(x) = x(x+2)(x-1)^2$. The graph of $g''(x)$ is shown below. For what values of x does g have a point of inflection? [3 points]



   a. –2 and 0 only b. 0 and 1 only c. –1 and 1 only

   b. d. –2, –1 and 0 e. –2, 0 and 1

Part 2: Calculators required for some of these questions

1. The position of a particle moving on the *x*-axis is given by $x(t) = t^2 - 5\sin(0.8t)$. For what value of *t* is the particle furthest left? [3 points]

   a. –1.177

   b. –1.287

   c. 1.287

   d. 1.177

   e. 1.391

2. For all *x* in the interval [4, 7] the function *f* is increasing and concave down. Which of the following could be a table of value for *f* ? [3 points]

a.

x	$f(x)$
4	11
5	13
6	16
7	20

b.

x	$f(x)$
4	11
5	15
6	18
7	20

c.

x	$f(x)$
4	17
5	13
6	10
7	8

d.

x	$f(x)$
4	17
5	15
6	12
7	8

e.

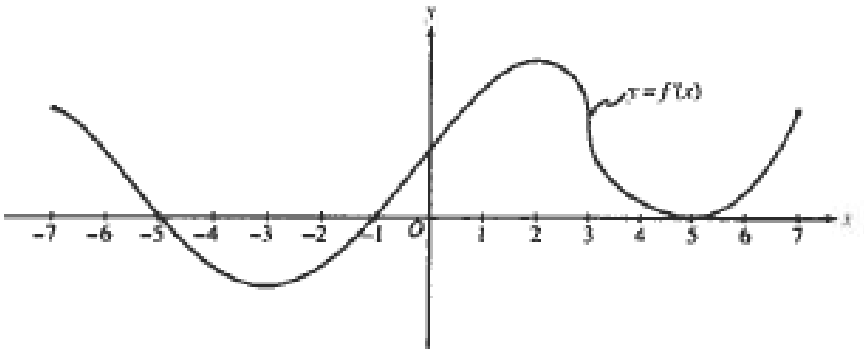
x	$f(x)$
4	17
5	14
6	11
7	8

3. A function has a derivative given by $f'(x) = \dfrac{\sqrt{x}}{x^2 + x + 1}$. What is the *x*-coordinate of the inflection point of *f* ? [3 points]

   a. –0.768 b. 0 c. 0.434

   d. 1 e. The graph of *f* has no inflection point.

4. [9 points]



The figure above shows the graph of f' the derivative of *f*, for $-7 \le x \le 7$. The graph of f' has horizontal tangent lines at $x = -3$, $x = 2$ and $x = 5$ and a vertical tangent line at $x = 3$.

   a. Find all values of *x*, for $-7 < x < 7$, at which *f* attains a relative minimum. Justify your answer.

   b. Find all values of *x*, for $-7 < x < 7$, at which *f* attains a relative maximum. Justify your answer.

   c. Find all values of *x*, for $-7 < x < 7$, at which $f''(x) < 0$.

Answers:

Part 1:

1. Increasing $\left[0,\frac{2\pi}{3}\right]$ and $\left[\frac{4\pi}{3},2\pi\right]$; decreasing $\left[\frac{2\pi}{3},\frac{4\pi}{3}\right]$ Allow open intervals for full credit.
2. [–1,5] or (–1, 5)
3. $y=-x+1$ The tangent lies above the graph since the graph is concave down at this point. Justify by showing that $f''(x)=-\cos(x)$ and $f''(0)=-1<0$
4. C. (0, 2)
5. A. –2 and 0 only

Part 2:

1. D 1.177 find the zero of the derivative on your calculator.
2. B only A and B are increasing. B is increasing at a decreasing rate so it is concave down.
3. C graph the given derivative. It's one extreme values is the location of the functions point of inflection.
4. Grade according to the scoring standard for 2000 AB 3. Use the PDF version for a better copy of the standard.

(a) $x=-1$ $f'(x)$ changes from negative to positive at $x=-1$	2 { 1 : answer 1 : justification
(b) $x=-5$ $f'(x)$ changes from positive to negative at $x=-5$	2 { 1 : answer 1 : justification
(c) $f''(x)$ exists and f' is decreasing on the intervals $(-7,-3)$, $(2,3)$, and $(3,5)$	2 { 1 : $(-7,-3)$ 1 : $(2,3)\cup(3,5)$
(d) $x=7$ The absolute maximum must occur at $x=-5$ or at an endpoint. $f(-5)>f(-7)$ because f is increasing on $(-7,-5)$ The graph of f' shows that the magnitude of the negative change in f from $x=-5$ to $x=-1$ is smaller than the positive change in f from $x=-1$ to $x=7$. Therefore the net change in f is positive from $x=-5$ to $x=7$, and $f(7)>f(-5)$. So $f(7)$ is the absolute maximum.	3 { 1 : answer 1 : identifies $x=-5$ and $x=7$ as candidates – or – indicates that the graph of f increases, decreases, then increases 1 : justifies $f(7)>f(-5)$